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PROPAGATION OF KELVIN WAVES FROM A CHANNEL INTO A SEMIBOUNDED TANK*

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The problem of propagation of Kelvin waves from a channel into a semibounded tank is considered. An exact solution of the problem is constructed using the Wiener–Hopf method. The solution is analyzed asymptotically and numerically. The Wiener–Hopf method was used in /1,2/ to solve the problem of diffraction of the Kelvin waves in tanks bounded by the infinite and semiinfinite parallel walls. Below a generalized method of matching /3/ is used to solve the problem of diffraction of Kelvin waves in the case when the walls confining the fluid meet at the right angles.

1. Formulation of the problem. Consider a system consisting of a channel and a tank of finite depth h, rotating anticlockwise with angular velocity ω. The channel and tank walls are described by the equations $x<0, |y|=a$ and $x=0, |y|>a$ respectively (Fig.1). We consider the harmonic wave motions of the fluid surface, the elevations of which can be written in the form $\xi(x, y)\cdot\exp(-i\sigma t)$ where σ is the frequency of these oscillations. Let us consider the case $\sigma>2\omega$. In the linear theory of long surface waves /4/ the function $\xi(x, y)$ represents the solution of the wave equation

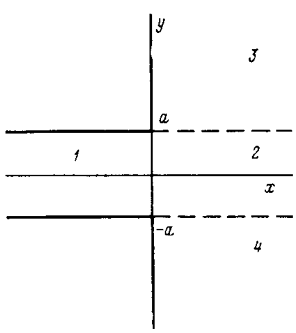

Fig.1

$$(\Delta+\varkappa^2)\xi(x, y)=0,\quad \varkappa^2=\frac{\sigma^2-4\omega^2}{gh},\quad \Delta=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$$

where g is acceleration due to gravity and Δ the two-dimensional Laplace operator. Let a Kelvin wave of unit amplitude propagate through the channel

$$\xi_0(x, y)=\exp[i\varkappa x-l\eta\varkappa(y+a)],\quad l=2\omega/\sigma,\quad \eta=(1-l^2)^{-1/2}$$

We shall study the fluid surface elevations in the system, generated by this wave during its diffraction at the open end of the channel. We shall call the channel region 1, and divide the tank into the regions 2,3 and 4 as shown in Fig.1. We write the total amplitude of the elevations in the channel $(x<0, |y|<a)$ in the form $\xi_0+\xi_1$ where ξ_0 denotes the incident, and ξ_1 the diffracted waves. We denote the unknown elevation amplitude in the extension of the channel, i.e. in region 2 $(x>0, |y|<a)$ by ξ_2, and in the regions 3 $(x>0,\ y>a)$ and 4 $(x>0,\ y<-a)$ by ξ_3 and ξ_4 respectively. We have the following boundary value problem for the unknown functions ξ_j $(j=1, 2, 3, 4)$:

$$(\Delta+\varkappa^2)\xi_j(x, y)=0 \tag{1.2}$$

$$v_1(x, a)=v_1(x,-a)=0,\quad x<0 \tag{1.3}$$

$$u_3(0, y)=0,\ y>a;\quad u_4(0, y)=0,\ y<-a$$

$$\begin{aligned}
&\xi_3(x, a+0)=\xi_2(x, a-0),\ x>0 \\
&\xi_4(x,-a-0)=\xi_2(x,-a+0),\ x>0 \\
&\xi_0(0, y)+\xi_1(0, y)=\xi_2(0, y),\ |y|<a \\
&u_3(x, a+0)=u_2(x, a-0),\ x>0 \\
&u_4(x,-a-0)=u_2(x,-a+0),\ x>0 \\
&u_0(0, y)+u_1(0, y)=u_2(0, y),\ |y|<a
\end{aligned} \tag{1.4}$$

Here u_j and v_j $(j=0, 1, 2, 3, 4)$ are the components of the velocity of motion of the fluid parallel to the x- and y-axes, and connected with $\xi_j(x, y)$ by the relations

$$u_j(x, y)=\frac{\sigma}{\varkappa^2 h}\left(-i\frac{\partial}{\partial x}+l\frac{\partial}{\partial y}\right)\xi_j(x, y),\quad v_j(x, y)=-\frac{\sigma}{\varkappa^2 h}\left(l\frac{\partial}{\partial x}+i\frac{\partial}{\partial y}\right)\xi_j(x, y) \tag{1.5}$$

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The conditions (1.3) represent the boundary conditions at the walls of the system, (1.4) are the conditions of continuity of the elevations and the velocity components x. Finally, the diffracted waves must satisfy the condition at the edge /5/

$$\xi_j \sim r^{2/3}, \quad r \to 0, \quad r = \sqrt{x^2 + (y \pm a)^2} \tag{1.6}$$

and the condition of radiation, and the solution at infinity must contain the divergent waves only. It can be shown that the problem (1.2) — (1.6) has a unique solutions in the class of bounded functions.

2. System of functional equations. To solve the problem (1.2) — (1.6) we assume that the wave number $\varkappa$ has a small positive imaginary part which will be made to tend to zero in the final results. The introduction of the imaginary part to $\varkappa$ corresponds to the assumption of energy dissipation in the fluid. We shall seek the diffracted field of elevations in the channel ξ_1 in the form of superposition of the following waves /2/: the Kelvin wave reflected from the open channel end, a finite number of progressive waves propagating through the channel in the negative direction of the x-axis, and an infinite number of waves decaying exponentially in the direction along the channel away from its open end

$$\xi_1(x, y) = A_r \exp[-i\eta\varkappa x + l\eta\varkappa(y - a)] + \sum_{k=1}^{\infty} [R_k \sin(\gamma_k y - \varphi_k) + T_k \cos(\gamma_k y - \varphi_k)] \exp(-i\alpha_k x) \tag{2.1}$$

$$\gamma_k = \frac{\pi k}{2a}, \quad \alpha_k = \sqrt{\varkappa^2 - \gamma_k^2}, \quad \operatorname{Im}\alpha_k > 0, \quad \sin\varphi_k = \frac{\gamma_k}{\sqrt{\gamma_k^2 + \alpha_k^2 l^2}}, \quad \cos\varphi_k = \frac{\alpha_k l}{\sqrt{\gamma_k^2 + \alpha_k^2 l^2}}$$

The unknown amplitudes R_k are zero at $k = 2, 4, 6, \ldots$, and the amplitudes T_k are zero at $k = 1, 3, 5, \ldots$. The number of progressive waves in the expression (2.1) coincides with the value of the integral part of $2\varkappa a/\pi$.

Let us introduce the auxilliary functions $U_j(y, \alpha)$ $(j = 2, 3, 4)$ of the complex variable α according to the formulas

$$U_j(y, \alpha) = \int_0^{+\infty} u_j(x, y) \exp(i\alpha x)\, dx \tag{2.2}$$

The properties of the Fourier transform in a complex plane imply that $U_j(y, \alpha)$ are regular functions of the complex variable α in the half-plane $\operatorname{Im}\alpha > 0$. To obtain the equations satisfied by these functions, we multiply both parts of the wave equation (1.1) written for $u_j(x, y)$ by $\exp(i\alpha x)$ and integrate it with respect to x from 0 to $+\infty$, with the condition of radiation taken into account. This yields

$$\left(\frac{\partial^2}{\partial y^2} + \gamma^2\right) U_j(y, \alpha) = \frac{\partial u_j}{\partial y}(0, y) - i\alpha u_j(0, y), \quad \gamma = \sqrt{\varkappa^2 - \alpha^2} \tag{2.3}$$

where the root branch is chosen so that $\operatorname{Im}\gamma > 0$. We shall seek the functions $U_j(y, \alpha)$ in the form of a sum

$$U_j(y, \alpha) = U_{js}(y, \alpha) + U_{ja}(y, \alpha) \tag{2.4}$$

Here and henceforth we shall denote by s and a the functions even and odd with respect to y. We shall give the method of solving (2.3) for the even component and only quote the final result for the odd component.

Let us turn our attention to region 3. Taking into account the second boundary condition (1.3), we shall write the equation for the function $U_{3s}(y, \alpha)$ in the form

$$\left(\frac{\partial^2}{\partial y^2} + \gamma^2\right) U_{3s}(y, \alpha) = \frac{\partial u_{3s}}{\partial y}(0, y) \tag{2.5}$$

To eliminate the unknown function appearing in the right-hand side of this equation, we replace α by $-\alpha$ in (2.5) and subtract the result from (2.5). This yields an equation, the solution of which satisfies the condition of radiation and has the form

$$U_{3s}(y, \alpha) - U_{3s}(y, -\alpha) = F(\alpha) \exp(i\gamma y) \tag{2.6}$$

where $F(\alpha)$ is an unknown function of the complex variable α. Differentiating (2.6) with respect to y and eliminating $F(\alpha)$, we obtain a relation connecting the functions with their derivatives

$$U_{3s}(y,\alpha)-U_{3s}(y,-\alpha)=\frac{1}{i\gamma}\left[\frac{\partial U_{3s}}{\partial y}(y,\alpha)-\frac{\partial U_{3s}}{\partial y}(y,-\alpha)\right]$$

We use the first formula of (1.5) to find the x-component of the velocity in the channel $u_1(x, y)$, and differentiate it with respect to y. Next we put $x = 0$ in the expressions for $u_1(x, y)$ and $\partial u_1(x, y)/\partial y$, and write each of these expressions in the form of a sum of two terms, even with respect to y and odd. Substitution of the even terms into the right-hand side of (2.3) for $U_{2s}(y, \alpha)$ yields an equation, whose solution, even with respect to y, has the form

$$\begin{aligned}
U_{2s}(y,\alpha) &= -\frac{iK_i}{\eta\varkappa+\alpha}\operatorname{ch}k_y y+\frac{iK_r}{\eta\varkappa-\alpha}\operatorname{ch}k_y y-\\
&\quad i\sum_{k=1}^{\infty}\frac{A_k}{\alpha_k-\alpha}\cos\gamma_k y+B(\alpha)\cos\gamma y\\
K_i &= -\frac{\varkappa}{\eta}\exp(-l\eta\varkappa a),\quad K_r=A_rK_i,\quad k_y=l\eta\varkappa\\
A_k &= \begin{cases}(-\alpha_k\sin\varphi_k+l\gamma_k\cos\varphi_k)R_k, & k=1,3,5,\ldots\\ (-\alpha_k\cos\varphi_k-l\gamma_k\sin\varphi_k)T_k, & k=2,4,6,\ldots\end{cases}
\end{aligned}\tag{2.7}$$

and contains an unknown function $B(\alpha)$. Let us differentiate (2.7) with respect to y, and eliminate $B(\alpha)$. We replace α by $-\alpha$ in the resulting expression, and subtract the result from the original expression. Setting $y = a$, taking into account the fourth condition of (1.4) and the relation (2.6), we arrive at the following functional equation:

$$\begin{aligned}
U_{2s}(a,\alpha)-U_{2s}(a,-\alpha) &= P_sY_0(\alpha)+Q_sX_0(\alpha)+\\
&\quad i\sum_{k=1}^{\infty}A_k\cos\gamma_k aY_k(\alpha)-\sum_{k=1}^{\infty}A_k\gamma_k\sin\gamma_k aX_k(\alpha)\\
P_s &= \frac{i\varkappa}{2\eta}[1+\exp(-2l\eta\varkappa a)](1-A_r)\\
Q_s &= \frac{l\varkappa^2}{2}[1-\exp(-2l\eta\varkappa a)](1-A_r)\\
X_k &= \frac{\alpha}{\gamma(\alpha^2-\alpha_k^2)}[1+\exp(2i\gamma a)]\\
Y_k(\alpha) &= \frac{\alpha}{\alpha^2-\alpha_k^2}[1-\exp(2i\gamma a)],\quad \alpha_0=\eta\varkappa
\end{aligned}\tag{2.8}$$

Repeating the above process for the odd term of the solution yields the second functional equation

$$\begin{aligned}
U_{2a}(a,\alpha)-U_{2a}(a,-\alpha) &= P_aF_0(\alpha)+Q_aG_0(\alpha)+\\
&\quad i\sum_{k=1}^{\infty}B_k\sin\gamma_k aF_k(\alpha)+\sum_{k=1}^{\infty}B_k\gamma_k\cos\gamma_k aG_k(\alpha)\\
P_a &= \frac{i\varkappa}{2\eta}[1-\exp(-2l\eta\varkappa a)](1+A_r)\\
F_k(\alpha) &= \frac{\alpha}{\alpha^2-\alpha_k^2}[1+\exp(2i\gamma a)]\\
Q_a &= -\frac{l\varkappa^2}{2}[1+\exp(-2l\eta\varkappa a)](1+A_r)\\
G_k(\alpha) &= \frac{\alpha}{\gamma(\alpha^2-\alpha_k^2)}[1-\exp(2i\gamma a)]\\
B_k &= \begin{cases}(-\alpha_k\cos\varphi_k+l\gamma_k\sin\varphi_k)R_k, & k=1,3,5,\ldots\\ (-\alpha_k\sin\varphi_k-l\gamma_k\cos\varphi_k)T_k, & k=2,4,6,\ldots\end{cases}
\end{aligned}\tag{2.9}$$

3. Reduction of the problem to a system of linear algebraic equations. Numerical and asymptotic analysis. The functional equations (2.8) and (2.9) can be reduced to an infinite system of linear equations. We shall show this using the even term of the solution as an example.

Let us separate the functions $X_k(\alpha)$ and $Y_k(\alpha)$ into terms regular in the upper and lower half-plane, denoted by the plus and minus subscript respectively

$$X_k(\alpha) = X_{k+}(\alpha) - X_{k-}(\alpha),\ Y_k(\alpha) = Y_{k+}(\alpha) - Y_{k-}(\alpha)\tag{3.1}$$

We can expand (3.1) using the Hilbert transformation formulas /6/. Thus e.g. we have

$$X_{k+}(\alpha) = \frac{1}{2\pi i}\int_{-\infty}^{+\infty}\frac{X_k(\beta)\,d\beta}{\beta-\alpha}$$

and $X_{k-}(\alpha) = -X_{k+}(-\alpha)$, since $X_k(-\alpha) = -X_k(\alpha)$.

As a result of this expansion, we can separate in (2.8) the functions regular in different half-planes of the complex variable α, and according to the Liouville theorem we obtain

$$U_{2s}(a,\alpha) - P_s X_{0+}(\alpha) + Q_s Y_{0+}(\alpha) - \sum_{k=1}^{\infty}{}' A_k\gamma_k \sin\gamma_k a X_{k+}(\alpha) - i\sum_{k=2}^{\infty}{}' A_k \cos\gamma_k a Y_{k+}(\alpha) = P(\alpha) \tag{3.2}$$

where $P(\alpha)$ is a polynomial, and a prime accompanying the summation sign denotes the summation through unity.

The condition at the edge (1.6) must be satisfied for the choice of the solution to be unique, and this implies the fulfilment of the following asymptotic estimates:

$$U_{2s}(a,\alpha) \sim \alpha^{-2/3}\ (|\alpha| \to +\infty);\quad R_k,\ \ T_k \sim k^{-5/3}\ (k \to +\infty)$$

The above relations help us to estimate the asymptotics of the left-hand part of the equation (3.2), and to establish that $P(\alpha) \equiv 0$.

It is easy to obtain the relations connecting the unknown amplitudes with the values of the function $U_{2s}(a,\alpha)$ in the points α_m $(m = 1, 3, 5, \ldots)$. Indeed, the right-hand part of the expression for the function $\partial U_{2s}(a,\alpha)/\partial y$ regular for $\operatorname{Im}\alpha > 0$ should have no poles at $\alpha = \alpha_m$ $(m = 1, 3, 5, \ldots)$. This yields

$$U_{2s}(a,\alpha_m) = -\frac{\alpha_m a}{\gamma_m}\sin\gamma_m a A_m - i\frac{K_i \operatorname{ch} k_y a}{\eta\varkappa+\alpha_m} - i\frac{K_r \operatorname{ch} k_y a}{\eta\varkappa-\alpha_m} - i\sum_{k=2}^{\infty}{}' \frac{A_k\cos\gamma_k a}{\alpha_k-\alpha_m}\qquad (m = 1, 3, 5, \ldots) \tag{3.3}$$

from which we obtain the following linear algebraic equations for the unknown amplitudes A_r and A_k $(k = 1, 2, 3, \ldots)$:

$$\begin{gathered}\left\{\frac{i\varkappa}{2\eta}[1+\exp(-2l\eta\varkappa a)]\left[X_{0+}(\alpha_m) - \frac{1}{\alpha_m-\eta\varkappa}\right] + \frac{l\varkappa^2}{2}[1-\exp(-2l\eta\varkappa a)]\,Y_{0+}(\alpha_m)\right\}A_r + \\ \sum_{k=1}^{\infty}{}' \gamma_k\sin\gamma_k a X_{k+}(\alpha_m)A_k - \frac{i\alpha_m a\sin\gamma_m a}{\gamma_m}A_m - i\sum_{k=2}^{\infty}{}'\cos\gamma_k a\left[Y_{k+}(\alpha_m) + \frac{1}{\alpha_k-\alpha_m}\right]A_k = \\ \frac{l\varkappa^2}{2}[1-\exp(-2l\eta\varkappa a)]\,Y_{0+}(\alpha_m) + \frac{i\varkappa}{2\eta}[1+\exp(-2l\eta\varkappa a)]\left[X_{0+}(\alpha_m) - \frac{1}{\alpha_m+\eta\varkappa}\right]\ (m = 1, 3, 5, \ldots)\end{gathered} \tag{3.4}$$

In the same manner we obtain the linear algebraic equations for the unknown amplitudes A_r and B_k $(k = 1, 2, 3, \ldots)$

$$\begin{gathered}\left\{\frac{i\varkappa}{2\eta}\left[1-\exp(-2l\eta\varkappa a)\right]\left[\frac{1}{\alpha_m-\eta\varkappa} - F_{0+}(\alpha_m)\right] - \frac{l\varkappa^2}{2}[1+\exp(-2l\eta\varkappa a)]\,G_{0+}(\alpha_m)\right\}A_r + \\ i\sum_{k=1}^{\infty}{}'\sin\gamma_k a\left[F_{k+}(\alpha_m) + \frac{1}{\alpha_k-\alpha_m}\right]B_k + \sum_{k=2}^{\infty}{}'\gamma_k\cos\gamma_k a G_{k+}(\alpha_m)B_k - \frac{i\alpha_m a\cos\gamma_m a}{\gamma_m}B_m = \\ \frac{i\varkappa}{2\eta}[1-\exp(-2l\eta\varkappa a)]\left[F_{0+}(\alpha_m) - \frac{1}{\alpha_m-\eta\varkappa}\right] + \frac{l\varkappa^2}{2}[1+\exp(-2l\eta\varkappa a)]\,G_{0+}(\alpha_m)\quad (m = 2, 4, 6\ldots)\end{gathered} \tag{3.5}$$

The functions $F_{k+}(\alpha)$ and $G_{k+}(\alpha)$ regular in the upper half-plane are found using the Hilbert transform formulas.

Thus the initial problem of diffraction of Kelvin waves has been reduced to that of solving an infinite system of linear algebraic equations for the amplitudes of the waves generated in the channel. The infinite system of linear equations (3.4)—(3.5) was solved using the numerical truncation method. To control the accuracy, the values of $|R_k|\, k^{3/2}$, $|T_k|\, k^{3/2}$ which tended sufficiently rapidly to constant values were printed out.

Fig.2 depicts the dependence of the modulus of A_r on $\varkappa a$, and Figs.3 and 4 show the dependence of the amplitude moduli of the first three even and odd progressive waves on $\varkappa a$. The graphs show salient points, characteristic for the diffraction problems, at

$$\varkappa a = \pi n/2 \tag{3.6}$$

In Fig.3 the salient points are observed at $n = 3, 5, \ldots$, and in Fig.4 at $n = 4, 6, \ldots$. The salient points are caused by the rearrangement of the wave motions at the instant of emergence of a new progressive wave. The phenomenon is known in electrodynamics /7/ and nuclear physics /8/ under the name of threshold effect. In /2/ the authors show the salient points in the graphs of the progressive waves in a channel for the same (3.6) values of $\varkappa a$.

In the present case we must, in order to satisfy the boundary conditions at the tank walls, take into account simultaneously the wave motions in the tank generated by the waves of both symmetries present in the channel. It follows that, unlike in /2/, the even and odd waves are interdependent. This can easily be observed in Figs.3 and 4. Indeed, the amplitudes of the waves associated with each symmetry exhibit the characteristic local minima at the instant of emergence of a wave of the opposite symmetry. At the same time, the amplitude of the reflected Kelvin wave undergoes characteristic salient points at the instants of emergence of the progressive waves of either symmetry.

In conclusion we use the region 4 $(x > 0,\ y < -a)$ to show the method of obtaining an expression for the elevations from the known solution of the linear system (3.4)—(3.5). We denote the right-hand parts of the equations (3.4) and (3.5) by $F_s(\alpha)$ and $F_a(\alpha)$ respectively. For $U_4(y, \alpha)$ the following relation holds:

$$\left(\frac{\partial^2}{\partial y^2} + \gamma^2\right) U_4(y, \alpha) = \frac{\partial u_4}{\partial y}(0, y)$$

Its solution, with the choice of the branch of the root γ taken into account, has the form

$$U_4(y, \alpha) - U_4(y, -\alpha) = D(\alpha) \exp(-i\gamma y) \tag{3.7}$$

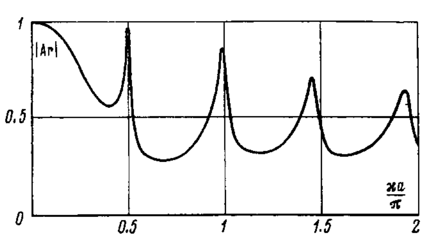

Fig.2

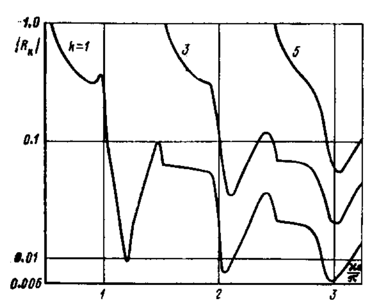

Fig.3

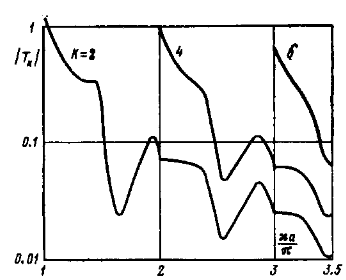

Fig.4

Assuming $y = -a$ in (3.7) and taking into account the matching conditions (1.4), we obtain the function $D(\alpha)$ in explicit form

$$D(\alpha) = [F_s(\alpha) - F_a(\alpha)] \exp(-i\gamma a)$$

Extending now the definition of the velocity component $u_4(x, y)$ to the region $x < 0,\ y < -a$ in accordance with the formula $u_4(x, y) = -u_4(-x, y)$ and inverting the Fourier transform (2.2), we arrive at the integral representation of the x-component of the velocity in region 4

$$u_4(x, y) = \frac{1}{4\pi} \int_{-\infty}^{+\infty} [F_s(\alpha) - F_a(\alpha)] \exp[-i\alpha x - i\gamma(y + a)]\, d\alpha \tag{3.8}$$

Integrating (1.12) for $j = 4$ and taking into account (3.8), we obtain the formula for the elevations

$$\xi_4(x, y) = \frac{1}{4\pi} \int_{-\infty}^{+\infty} \frac{F_s(\alpha) - F_a(\alpha)}{\alpha + i\gamma l} \exp[-i\alpha x - i\gamma(y + a)]\, d\alpha \tag{3.9}$$

The above integral cannot be computed in explicit form, but the pole of the integrand at the point $\alpha = -il\eta\varkappa$ corresponds to a Kelvin wave propagating along the wall $x = 0$, $y < -a$ in the negative direction of the y-axis. Calculating the residue at this point we obtain the amplitude of the Kelvin wave

$$\xi_k = -\frac{i\eta^2}{2}[F_s(-il\eta\varkappa) - F_a(-il\eta\varkappa)] \exp[-l\eta\varkappa x - i\eta\varkappa(y + a)]$$

Estimating now the integral (3.9) with help of the method of steepest descent, we obtain the following expression for the elevations in region 4 at large distances from the channel entry $(\varkappa r \gg 1)$:

$$\xi_4(r, \theta) \sim \frac{1}{4\pi\sqrt{\varkappa r}} \exp[i(\varkappa r - \pi/4)] \times \varkappa\cos\theta\, [F_s(-\varkappa\sin\theta) - F_a(-\varkappa\sin\theta)] \tag{3.10}$$

The polar coordinates r, θ with the center at the point $x = 0$, $y = -a$ were introduced according to the formula $x = r\sin\theta$, $y + a = -r\cos\theta$. We see from (3.10) that the elevations at infinity represent divergent, damped cylindrical waves with the angular distribution of amplitude

$$|\cos\theta\, [F_s(-\varkappa\sin\theta) - F_a(-\varkappa\sin\theta)]|$$

In conclusion we note that the length and time units were chosen so that $\sigma/(\varkappa^2 h) = 1$.

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